Cooperative Games

Lecture 11: Games with externality and a short survey of the multiagent systems litterature.

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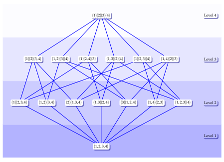
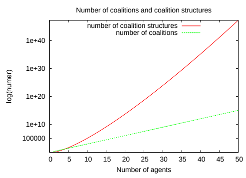
Today

- Search of the optimal coalition structure (AI topic)
- Games with a priori union (a different interpretation of a coalition structure)
- Games with externalities
- Issues related to the formation of coalitions.

Search of the Optimal Coalition Structure





The difficulty of searching for the optimal CS is the large search space.

How to distribute the computation of all the coalition values?

- goal is to minimize computational time
  Computing the value of a coalition can be hard: ex solving a TSP
- load balancing: distribute coalitions of every size equally among the agents coalitions.

but agents may have different compuatational speed

A naive approach does not avoid redundancy and may have a high communication complexity.
The current best algorithm works by sharing the computation of coalition of the same size between all the agents.

O. Shehory and S. Kraus. **Methods for task allocation via agent coalition formation**. *Artificial Intelligence*, 1998

T. Rahwan and N. Jennings. **An algorithm for distributing coalitional value calculations among cooperating agents**, *Artificial Intelligence*, 2007

Search of the Optimal Coalition Structure

Any time algorithm: search through the space of CSs.

- algorithms provide a bound to the optimal.
  It is necessary to visit a least 2^{n-1} CSs, which corresponds to the first two levels of the lattice.
  ⤳Let $\mathcal{S}'$ be the best CS in the first two levels, then $v(CS^\star) \leqslant n\hat{v}(\mathcal{S}')$.
- different techniques to visit the rest of the space.
- Best current algorithm is called IP for Interger Partition:
  - Integer Partition: ex [1,1,2] $\rightarrow$ space of coalition structures containing two singletons and a coalition of size 2.
  - Finding bounds for each subspace is easy. Ex: $\max_{S\in[1,1,2]} v(S) \leqslant \max_{\mathcal{C}\in 2^N, |\mathcal{C}|=1} v(\mathcal{C}) + \max_{\mathcal{C}\in 2^N, |\mathcal{C}|=1} v(\mathcal{C}) + \max_{\mathcal{C}\in 2^N, |\mathcal{C}|=2} v(\mathcal{C})$
  - IP uses the representation to efficiently prune part of the space and search the most promising subspaces.

Search of the Optimal Coalition Structure

Dynamic programming

Recall the superadditive cover $(N,\hat{v})$ of a game (N,v):

$$\hat{v}(\mathcal{C}) = \begin{cases} \max_{\mathcal{P}\in\mathscr{S}_\mathcal{C}} \left\{\sum_{T\in\mathcal{P}} v(T)\right\} \text{ for all } \mathcal{C}\subseteq N\setminus\emptyset \\ 0 \text{ for } \mathcal{C}=\emptyset \end{cases}$$

Lemma

For any $\mathcal{C}\subseteq N$, we have

$$\hat{v}(\mathcal{C}) = \max\left\{ \max\left\{ \hat{v}(\mathcal{C}')+\hat{v}(\mathcal{C}'') \;\middle|\; \begin{array}{c} \mathcal{C}'\cup\mathcal{C}''=\mathcal{C} \\ \mathcal{C}'\cap\mathcal{C}''=\emptyset \\ \mathcal{C}',\mathcal{C}''\neq\emptyset \end{array} \right\}, v(\mathcal{C}) \right\}.$$

Search of the Optimal Coalition Structure

Dynamic programming

To compute the optimal value, we can compute for $k=1\dots n$ $\hat{v}(\mathcal{C})$ for all subsets of $\mathcal{C}$ with $|\mathcal{C}|=k$.

- $k=1$**:** $\hat{v}(\mathcal{C}) = v(\mathcal{C})$
- $1<k\leqslant n$**:** consider all nomempty and disjoint $\mathcal{C}'$ $\mathcal{C}''$ such that $\mathcal{C}=\mathcal{C}'\cup\mathcal{C}''$
  (since $|\mathcal{C}'|$, $|\mathcal{C}''|\leqslant k-1$, we know the values of $\hat{v}(\mathcal{C}')$ and $\hat{v}(\mathcal{C}'')$)
  set $\hat{v}(\mathcal{C}) = \max\{v(C), \hat{v}(\mathcal{C}')+\hat{v}(\mathcal{C}'')\}$.

T. Sandholm, K. Larson, M. Andersson, O. Shehory, and F. Tohmé. **Coalition structure generation with worst case guarantees**, *Artificial Intelligence*, 1999.

T. Service and J. Adams **Constant factor approximation algorithm for coalition structure generation**, Autonomous Agents and MultiAgent Systems, 2010.

T. Rahwan, S.D. Ramchurn, N. Jennings, and A. Giovannucci. **An anytime algorithm for optimal coalition structure generation**, *Journal of Artificial Intelligence Research*, 2009.

Game with a priori unions

- We have considered that in a coalition structure (CS), each coalition works independently.
- One can also consider that the agents are forming the **grand coalition**, but they prefer to work or negotiate their payoff with a given group of agent.

Definition (Game with a priori unions)

A **game with a priori unions** is a triplet (N,v,S), where (N,v) is a TU game, and S is a particular CS. It is assumed that the grand coalition forms.

The Owen value

Owen uses the idea of the Shapley value but he does not consider all joining orders, only the ones that are **consistent** with the CS.

Definition (Consistency with a coalition structure)

A permutation π is *consistent* with a CS S when, for all $(i,j) \in \mathcal{C}^2$, $\mathcal{C} \in S$ and $l \in N$, $\pi(i) < \pi(l) < \pi(j)$ implies that $l \in \mathcal{C}$.

Definition (Owen value)

Given a game with a priori union (N,v,S), the **Owen value** $O_i(N,v,S)$ of agent i is given by

$$O_i(N,v,S) = \sum_{\pi \in \Pi_S(N)} \frac{mc(\pi)}{|\Pi_S(N)|}$$

Example of Owen value

$$N = \{1,2,3\}$$
$$v(\{1\}) = 0 \quad v(\{2\}) = 0 \quad v(\{3\}) = 0$$
$$v(\{1,2\}) = 90 \quad v(\{1,3\}) = 80 \quad v(\{2,3\}) = 70$$
$$v(\{1,2,3\}) = 120$$

$S_2 = \{\{1,2\},\{3\}\}$

	1	2	3
$1 \leftarrow 2 \leftarrow 3$	0	90	30
$1 \leftarrow 3 \leftarrow 2$		✘	
$2 \leftarrow 1 \leftarrow 3$	90	0	30
$2 \leftarrow 3 \leftarrow 1$		✘	
$3 \leftarrow 1 \leftarrow 2$	80	40	0
$3 \leftarrow 2 \leftarrow 1$	50	70	0
total	220	200	60
Owen value $O_i(N,v,S_1)$	55	50	15

$S_2 = \{\{1,3\},\{2\}\}$

	1	2	3
$1 \leftarrow 2 \leftarrow 3$		✘	
$1 \leftarrow 3 \leftarrow 2$	0	40	80
$2 \leftarrow 1 \leftarrow 3$	90	0	30
$2 \leftarrow 3 \leftarrow 1$	50	0	70
$3 \leftarrow 1 \leftarrow 2$	80	40	0
$3 \leftarrow 2 \leftarrow 1$		✘	
total	220	80	180
Owen value $O_i(N,v,S_2)$	55	20	45

One of the purpose of Game theory is to *"determine everything that can be said about coalitions between players, compensations between partners in every coaltion, mergers or fights between coalitions "...*

von Neumann and Morgenstern,
Theory of games and economic behaviour, 1944.

1- Which coalition will be formed?
2- How will the coalitional worth be shared between members?
3- How does the presence of other coalitions affect the incentives to cooperate?

Cooperative game theory has focused mainly on point 2.

Coalitional Games with externalities

- In a TU game (N,v), the valuation of a coalition depends only on the members, **not** on the other coalition present in the population.
- The value **can** depend on the other coalitions in the population
  - competitive firms
  - teams in sport
- ➪ valuation function for a coalition given a coalition structure (in a competitive setting) $v : 2^N \times \mathscr{S} \to \mathbb{R}$ Games **in partition function form**.
- ➪ valuation function for each agent given a coalition structure (ex: competitive supply chains) $v : N \times \mathscr{S} \to \mathbb{R}$. Games with **Valuations**.

Games in partition function form

A game in partition function is (N,v) where $v : 2^N \times \mathscr{S} \to \mathbb{R}$.

Definition (Superadditivity)

A partition function v is **superadditive** iff for any coalition structure π and any two coalitions S and T in π, $v(S \cup T, \pi \setminus \{S,T\} \cup \{S \cup T\}) \geqslant v(S,\pi) + v(T,\pi)$.

Definition (Monotonicity)

A partition function v is monotonic if for any two coalition S, $T \subseteq S$, for any partition π containing S, and any partition π' containing T such that π and π' coincide on $N \setminus S$, $v(S,\pi) \geqslant v(T,\pi')$.

Lemma

If a partition function is superadditive, then it is monotonic.

Games in partition function form

Definition (Positive and negative spillovers)

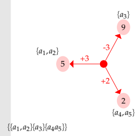
A partition function v exhibits

- **positive spillovers** if for any partition π and any two coalitions S and T in π $v(\mathcal{C}, \pi \setminus \{S,T\} \cup \{S \cup T\}) \geqslant v(C,\pi)$ for all coalitions $\mathcal{C} \neq S,T$ in π.
- **negative spillovers** if for any partition π and any two coalitions S and T in π $v(\mathcal{C}, \pi \setminus \{S,T\} \cup \{S \cup T\}) \leqslant v(C,\pi)$ for all coalitions $\mathcal{C} \neq S,T$ in π.

Some representation issues

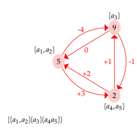
- externality from coalition formation $S = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\} \to S' = \{\mathcal{C}_1, \mathcal{C}_2 \cup \mathcal{C}_3\}$: externality from the merging of $\mathcal{C}2$ and $\mathcal{C}_3$ on coalition $\mathcal{C}_1$ is $v(\mathcal{C}_1,S) - v(\mathcal{C}_1,S')$
- value of **externality-free** of a coalition $\mathcal{C}$ is the value $v_{ef}(\mathcal{C}) = v(\mathcal{C},S)$ where $S \setminus \mathcal{C}$ is composed of singletons.
- total externality $\mathscr{T}$: Combined the externalities from a coalition formation process where all other coalitions start as a singleton: $v(\mathcal{C},S) = v_{ef}(\mathcal{C}) + \mathscr{T}(\mathcal{C},S)$

Inward externality



Assume that in $\{a_1,a_2\},\{a_3\},\{a_4,a_5\}$ without the influence, the payoff is 5,9 and 2 for each coalition.

Outward externality



Valuations

Assumption: Fixed rules of division appear naturally in many economic situations and in theoretical studies based on a two-stage procedure:

1- formation of the coalitions
2- payoff distribution

Definition (Valuation)

A **valuation** v is a mapping which associates to each coalition structure a payoff of individual payoff in $\mathbb{R}^n$.

Definition (Positive and negative spillovers)

A valuation v exhibits

- **positive spillovers** if for any partition π and any two coalitions S and T in π $v_i(\pi \setminus \{S, T\} \cup \{S \cup T\}) \geqslant v_i(\pi)$ for all players $i \notin S \cup T$.
- **negative spillovers** if for any partition π and any two coalitions S and T in π $v_i(\pi \setminus \{S, T\} \cup \{S \cup T\}) \leqslant v_i(\pi)$ for all players $i \notin S \cup T$.

Definition (Core stability)

A coalition structure π is **core stable** if there does not exist a group $\mathcal{C}$ of players a coalition structure π' that contains $\mathcal{C}$ such that $\forall i \in \mathcal{C}$, $v_i(\pi') > v_i(\pi)$.

Definition (α-core Stability)

A coalition structure π is **α-core stable** if there does not exist a group $\mathcal{C}$ of players and a partition $\pi'_{\mathcal{C}}$ such that, for all partition $\pi_{N\setminus\mathcal{C}}$ formed by external players, $\forall i \in \mathcal{C}$, $v_i(\pi'_{\mathcal{C}} \cup \pi_{N\setminus\mathcal{C}}) > v_i(\pi)$.

Definition (β-core Stability)

A coalition structure π is **β-core stable** if there does not exist a group $\mathcal{C}$ of players such that for all partitions $\pi_{N\setminus\mathcal{C}}$ of external players, there exists a partition $\pi_{\mathcal{C}}$ of $\mathcal{C}$ such that $\forall i \in \mathcal{C}$, $v_i(\pi_{\mathcal{C}} \cup \pi_{N\setminus\mathcal{C}}) > v_i(\pi)$.

The axioms of the Shapley value no longer guarantee the uniqueness of a value. Many authors proposed additional axioms.

- Pham Do and Norde focus on the *externality-free* value
- McQuillin focuses on CS for which the agents that have not joined the coalition are together in a coalition
- Myerson considers all joining orders.

{8}
{4,4} {3,5} {2,6} {1,7}
{2,3,3} {1,3,4} {1,2,5} {1,1,6}
{1,2,2,3} {2,2,2,2} {1,1,3,3} {1,1,2,4} {1,1,1,5}
{1,1,2,2,2} {1,1,1,2,3} {1,1,1,1,4}
{1,1,1,1,1,2,2} {1,1,1,1,1,1,3}
{1,1,1,1,1,1,2}
{1,1,1,1,1,1,1,1}

Issues studied in multiagent systems

Stability and Dynamic Environments

- Agents can enter and leave the environment at any time
- The characteristics of the agents may change with time

Extending some concepts to Open Environments.

N. Ohta, A. Iwasaki, M. Yokoo, K. Maruono, V. Conitzer, T. Sandholm, **A Compact Representation Scheme for Coalitional Games in Open Anonymous Environments** in *Proceedings of the Twenty First National Conference on Artificial Intelligence (AAAI-06)*, 2006.

M. Yokoo, V. Conitzer, T. Sandholm, N. Ohta, A. Iwasaki. **Coalitional games in open anonymous environments** in *Proceedings of the Twentieth National Conference on Artificial Intelligence (AAAI-05)*, 2005.

Uncertainty about Knowledge and Task

- Agents may not know some tasks.
- Agents may not know the valuation function, and may use Fuzzy sets to represent the coalition value.
- Expected values of coalitions are used instead of the valuation function.
- Approximation of valuation function: e.g., computing a value for a coalition requires solving a version of the traveling salesman problem and approximations are used to solve that problem.
- Agent do not know the cost incurred by other agents and may only estimate these costs.

Safety and Robustness

- Communication links may fail during the negotiation phase
- Payoff distribution close to the core but that does not need to restart the computation if an agent leaves the system.

Protocol Manipulation

A protocol may require that they disclose some private information.

- ➪ Avoid information asymmetry that can be exploited by some agents by using cryptographic techniques.
- ➪ Use computational complexity to protect a protocol.

Other types of manipulations:

- hiding skills
- using false names
- colluding

The traditional solution concepts can be vulnerable to false names and to collusion.
Study for some TU games and for weighted voting games.

Communication

Additional goals of the coalition formation: decreasing the time and the number of messages required to reach an agreement.

- learning may be used to decrease negotiation time.
- communication costs are represented in the characteristic function.
- analysis of the communication complexity of computing the payoff of a player with different stability concepts: they find that it is $\Theta(n)$ when the Shapley value, the nucleolus, or the core is used.

Long Term Vs Short Term

In general, a coalition is a short-lived entity that is *"formed with a purpose in mind and dissolve when that need no longer exists, the coalition ceases to suit its designed purpose, or critical mass is lost as agents depart"*.

- Long term coalitions, and in particular the importance of trust in this content.
- Repeated coalition formation under uncertainty using learning.

Overlapping Coalitions

Agents may simultaneously belong to more than one coalition

- Fuzzy approach
  - agents can be member of a coalition with a certain degree that represents the risk associated with being in that coalition.
  - agents have different degree of membership, and their payoff depends on this degree.
- Heuristic algorithms
- Game theoretical approach (overlapping cores)

Summary

- We considered the case where the value of a coalition does not depend only on the members of the coalition.
- Multiagent system research can propose solutions to many issues found in practice